



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.





1 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} k & 1 & 0 \\ 6 & 5 & 2 \\ -1 & 3 & -k \end{pmatrix},$$

where k is a real constant.

(a) Show that $\mathbf{A}$ is non-singular.

[3]

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(b) Given that $\mathbf{A}^{-1} = \begin{pmatrix} 3 & 0 & -1 \\ 1 & 0 & 0 \\ -\frac{23}{2} & \frac{1}{2} & 3 \end{pmatrix}$, find the value of k .

[2]

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2 The cubic equation $x^3 + 2x^2 + 3x + 1 = 0$ has roots α, β, γ .

(a) Find a cubic equation whose roots are $\alpha^2 + 1, \beta^2 + 1, \gamma^2 + 1$. [3]

This image shows a full page of primary-ruled paper. It consists of multiple horizontal rows, each defined by two parallel dashed lines. The lines are evenly spaced and extend across the entire width of the page, providing a guide for letter height and placement in handwriting practice. There are no margins, text, or other markings on the page.



- (b) Find the value of $(\alpha^2 + 1)^2 + (\beta^2 + 1)^2 + (\gamma^2 + 1)^2$. [2]

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- (c) Find the value of $(\alpha^2 + 1)^3 + (\beta^2 + 1)^3 + (\gamma^2 + 1)^3$. [2]

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3 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 7 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix **M** represents a sequence of two geometrical transformations in the x - y plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

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(b) Find the equations of the invariant lines, through the origin, of the transformation represented by **M**. [5]

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(c) Given that the area of triangle PQR is 35 cm^2 , find the area of triangle DEF . [2]

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1).$$

[5]

[illegible]



11

show that $S_n = \frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$. [6]

[illegible]

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- 5 The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i} + 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} - 2\mathbf{k})$ and $\mathbf{r} = -3\mathbf{i} + 4\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ respectively.

- (a) Find the shortest distance between l_1 and l_2 . [5]

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The plane Π_1 contains l_1 and is parallel to l_2 .

- (b) Obtain an equation of Π_1 in the form $px + qy + rz = s$. [2]

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Obtain an equation of Π_γ in the form $ax+by+cz=d$. [3]

[illegible]

(a) Show that C has no vertical asymptotes and state the equation of the horizontal asymptote. [2]

[illegible]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

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(d) Sketch $y^2 = \frac{x+1}{x^2+3}$, stating the coordinates of the stationary points and the intersections with the axes. [4]



(a) Sketch C and state the equation of the line of symmetry. [3]

(b) Find a Cartesian equation for C . [3]



[4]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The entire page is otherwise blank, with no text or other markings.

[6]

[illegible]

[illegible]



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